

Section One: Calculator-free

35% (52 Marks)

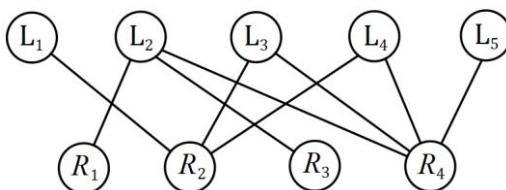
This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(5 marks)

Five university lecturers (L_1, L_2, L_3, L_4 and L_5) have been allocated four rooms (R_1, R_2, R_3 and R_4) to teach in. Only one lecturer will teach in a room at any time. Because some of the lecturers require specialist equipment, not all the rooms can be used by all the lecturers, as shown in the graph below.

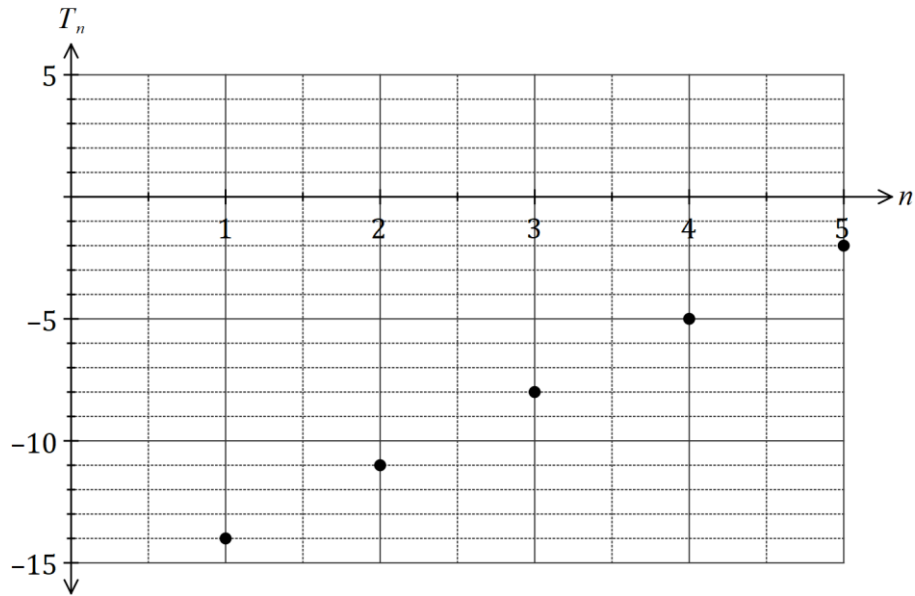


- (a) What is the name of such a graph shown above, where the vertices can be split into two groups so that each edge joins a vertex from one group to a vertex in the other group? (1 mark)
- (b) How many lecturers can use room R_4 ? (1 mark)
- (c) How many rooms can lecturer L_2 use? (1 mark)
- (d) Briefly explain whether
- (i) all five lecturers can teach at the same time? (1 mark)
- (ii) all four rooms could be in use at the same time? (1 mark)

Question 2

(4 marks)

The first five terms of an arithmetic sequence are shown on the graph below.



(a) Deduce a rule for the n^{th} term of this sequence. (2 marks)

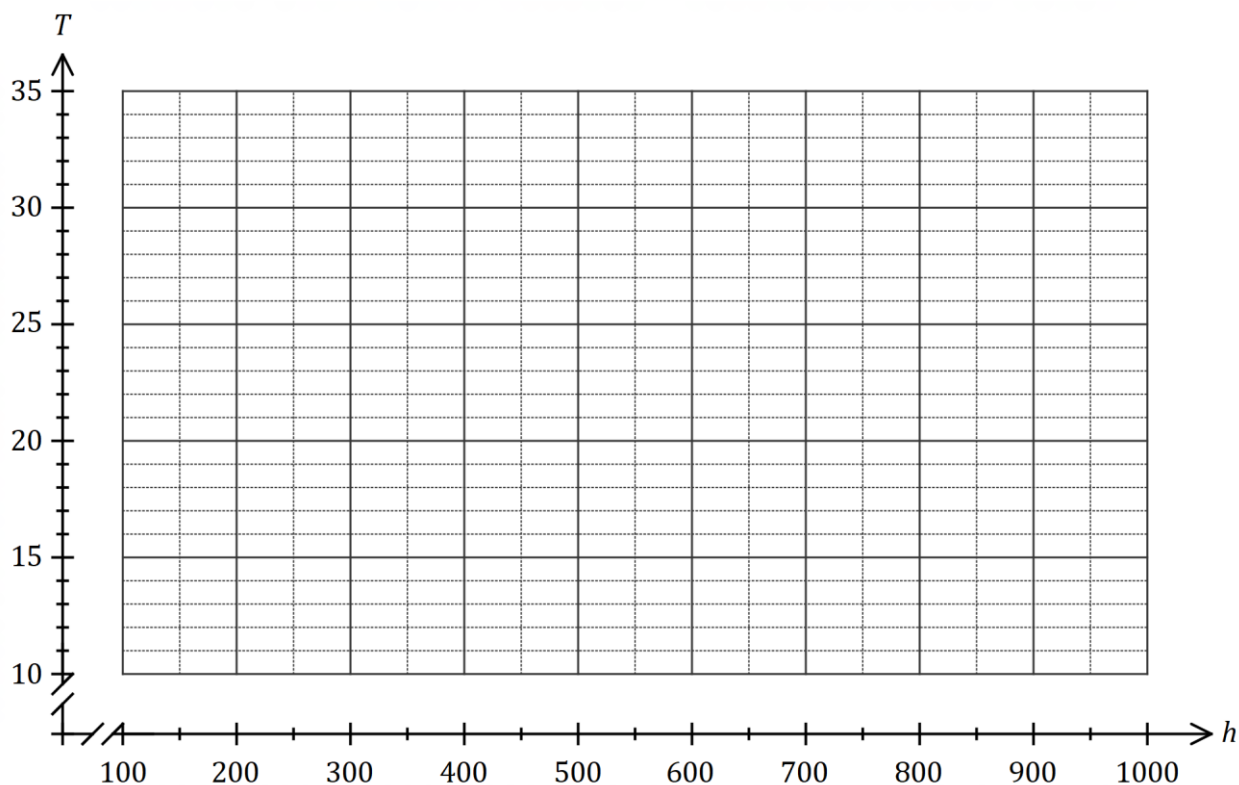
(b) Given that the k^{th} term of this sequence is 589, determine the value of k . (2 marks)

Question 3**(8 marks)**

The average maximum temperature, T °C, was recorded for ten weather stations, together with the altitude of the station, h metres. The data is shown in the table below.

Altitude, h	115	250	301	342	442	525	627	774	896	934
Temperature, T	26	25	28	29	25	22	23	22	16	16

- (a) Construct a scatterplot on the axes below that can be used to identify whether an association exists between altitude and temperature. (3 marks)



- (b) Describe the features of the scatterplot that indicate a negative, strong linear association exists between altitude and temperature. (2 marks)

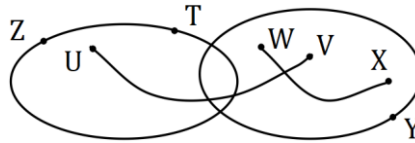
- (c) Estimate a value for

(i) the temperature at an altitude of 700 metres. (1 mark)

(ii) the value of the correlation coefficient between the two variables. (2 marks)

Question 4**(7 marks)**

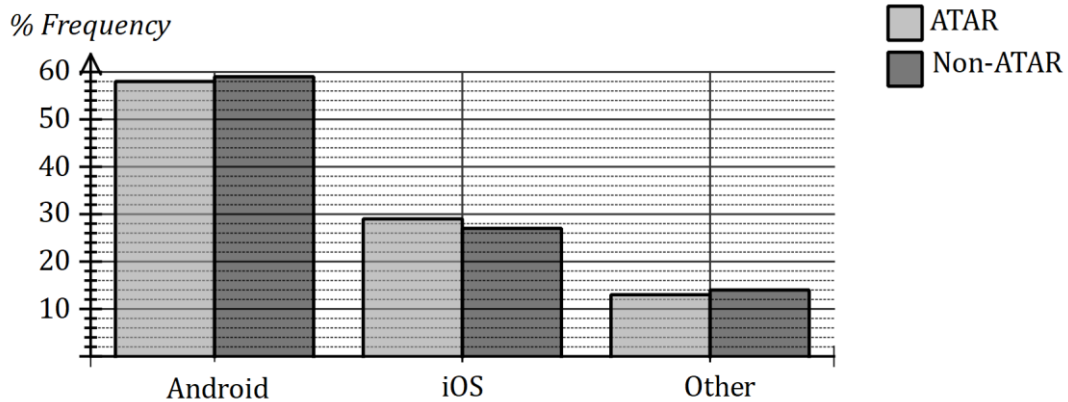
Graph G_1 is shown below, with vertices T, U, V, W, X, Y and Z .



- (a) In graph theory, a planar graph is a graph that can be drawn in the plane. Describe how to draw the edges of such a graph to clearly show that it is planar. (1 mark)
- (b) Redraw graph G_1 to clearly show that it is planar. (2 marks)
- (c) State the number of loops graph G_1 contains. (1 mark)
- (d) Show that Euler's formula does **not** apply to graph G_1 . (3 marks)

Question 5**(6 marks)**

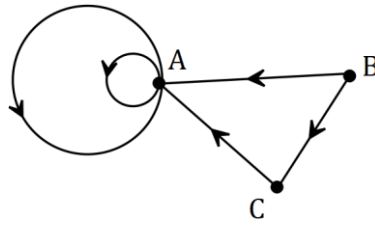
Mobile phone users who responded to a survey were asked which type of operating system their current phone used (Android, iOS or other) and whether they followed an ATAR or non-ATAR pathway at school. A breakdown of the results is shown in the graph below.



- (a) What percentage of the respondents who followed a non-ATAR pathway had a phone that used iOS? (1 mark)
- (b) Assuming that the survey results apply to the population in general,
- does knowing the operating system of a person's phone help you know the pathway they followed at school? Explain your answer. (2 marks)
 - does knowing the pathway a person followed at school help you know the type of operating system their phone has? (1 mark)
 - is there an association between a person's school pathway and type of operating system their phone has? Explain your answer. (2 marks)

Question 6**(7 marks)**

A digraph is shown below.

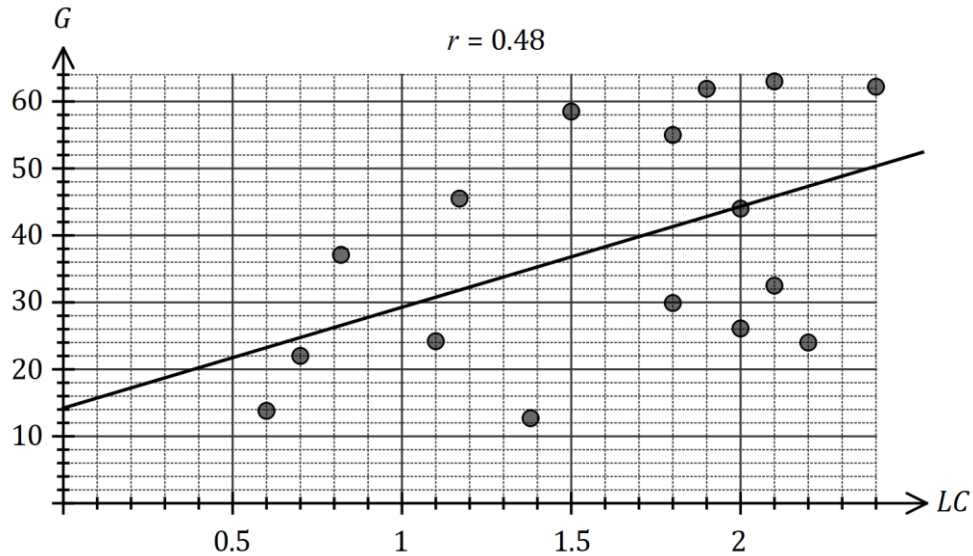


- (a) State, with justification, whether the digraph contains
- (i) a walk of length 8. (2 marks)
- (ii) a Hamiltonian path. (2 marks)
- (b) Using column and row headings in the order $A - B - C$, construct the adjacency matrix M for the digraph and explain why there is a lack of symmetry in the matrix. (3 marks)

Question 7

(7 marks)

A medical study measured the lipoprotein-cholesterol (LC) and ghrelin (G) levels of a group of patients. The results were displayed in the scatterplot below, together with the least-squares line of best fit and the correlation coefficient between the variables.



- (a) How many patients in the study with a lipoprotein-cholesterol level of less than 1.85 had a ghrelin level of more than 28? (1 mark)
- (b) Determine the upper and lower predicted ghrelin levels for patients with lipoprotein-cholesterol levels between 1.05 and 2.1. (2 marks)
- (c) Comment on the claim that a low lipoprotein-cholesterol level causes a patient to have a low ghrelin level. (2 marks)
- (d) State the number of patients in the study and comment on how the size of the study could influence any explanation for an association between the variables. (2 marks)

Question 8**(8 marks)**

A connected planar graph G_2 has three faces and four vertices.

(a) Determine the number of edges graph G_2 has. (2 marks)

(b) In each of the following, use the additional condition only within that part of the question.

(i) Draw graph G_2 so that it is simple. (2 marks)

(ii) Draw graph G_2 so that it contains a Eulerian trail. (2 marks)

(iii) Draw graph G_2 so that it contains a Hamiltonian path but not a Hamiltonian cycle. (2 marks)

Supplementary page

Question number: _____

